A Top Motions

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Abstract: Throughout the centuries, researchers derived dozens of gyroscope theories that did not describe the action of the undiscovered inertial torques. Only one torque the change in the angular momentum was represented by L. Euler. The physics of gyroscopic effects turned out to be many times harder than could imagine. Any rotating objects exposed by the action of the eight interacted torques operating by the centrifugal, common inertial, Coriolis forces, and the change in the angular momentum, which entailed by the ratio of the angular velocities of the rotating objects around their axes of motions. The interrelated inertial torques of the rotating object are constrained by the ratio of the angular velocities around axes that express the kinetic energies based on the principle of the conservation of mechanical energy This system of the five components formulates the fundamental principles of gyroscope theory which equations were corrected and represented are as follows:

- the torque of centrifugal and common inertial forces

$$T_{ct} = T_{in} = (2/9)\pi^2 J\omega\ _i$$

- the torque of Coriolis forces $T_{ct} = T_{in} = (8/9)J\omega\ _i$
- the torque of the change in angular momentum
- the ratio of the angular velocities around axes of the rotating object

$$\omega_y = \left[\frac{2\pi^2 + 8 + (2\pi^2 + 9)\cos\gamma}{2\pi^2 + 9 - (2\pi^2 + 8)\cos\gamma}\right]\omega_x$$

where wi is the angular velocity of the rotating object around axis i, w is its angular velocity around its own axis; J is the mass moment of inertia of the spinning disc; is of the angle of the axis of the rotating object inclination.

The new fundamental principles of gyroscope theory enable for formulating motions of any rotating objects. The top motions researchers could not describe exactly for the long-time. The acting forces on the tilted top spinning in a counter-clockwise direction are its weight, frictional force of the leg's tip at the point of contact the leg with the horizontal surface, and the system of inertial forces mentioned above (Fig. 1). The action of external and inertial torques leads to the decrease of the angular velocity of the top's spin. Hence, the values of inertial torques are decreasing gradually, while the precession velocities correspondingly increase. When the angular velocity of the top becomes smaller, the inertial torques become weaker. In this case, the tip of the top's leg describes a visible spiral curve with a decrease of its radius of curvature. This situation leads to the vertical approach of the top axis that manifests its stabilization

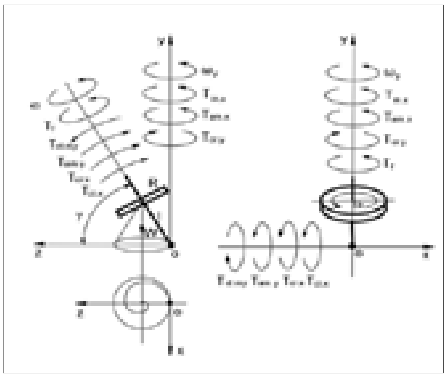

Figure 1. Torques acting on a spinning top.

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where W = Mg is the top's weight that is the external torque; M is its mass g is the gravity acceleration, *l* is the length of the leg; $F_{ct.mx}$ = $ml\omega_x^2 \sin\gamma\cos\gamma$ is the torque generated by the centrifugal force of the rotating top's centre mass around axis ox; T_f = Mgflcosγ is the torque generated by the frictional force acting on the tip and turns the top around it's the centre mass in the counter clockwise direction, *f* is the coefficient of the sliding friction between the leg and flat surfaces; Tct.my = Ml2cosγsinγωy2 is the torque generated by the centrifugal force of the rotating top's centre mass around axis oy; wy is the precession velocity of the top around axis oy; ω is the coefficient of the change in the value of the inertial torques; $T_{in.x}$, $T_{am.x}$, $T_{in.y}$ and $T_{am.y}$ are the precession torques of the common inertial forces and the change in the angular momentum acting around axis ox and oy respectively; Tcr.x, and Tcr.y are the Coriolis torques acting around axis ox and oy respectively;

$$\eta = 1 + \frac{9Mgf}{(2\pi^2 + 9)J\omega\ _x}$$

is the coefficient of the change in the precession torque Tam.y.

The corrected mathematical model in Euler's form for top motions around axes ox and oy (Fig.1) is represented by the following equations:

$$J_x \frac{d\omega_x}{dt} = Mgl\cos\gamma + M^{2}\cos\gamma\sin\gamma\left[\left(\frac{2\pi^2 + 8 + (2\pi^2 + 9)\cos\gamma}{2\pi^2 + 9 - (2\pi^2 + 8)\cos\gamma}\right)\omega_x\right]^2 -$$
$$\left[\left(\frac{2\pi^2 + 8}{9}\right) + \left(\frac{2\pi^2 + 8 + (2\pi^2 + 9)\cos\gamma}{2\pi^2 + 9 - (2\pi^2 + 8)\cos\gamma}\right)\right]J\omega\ _x -$$
$$\left(\frac{9}{2\pi^2 + 9}\right)\left(\frac{2\pi^2 + 8 + (2\pi^2 + 9)\cos\gamma}{2\pi^2 + 9 - (2\pi^2 + 8)\cos\gamma}\right)Mgf$$

$$J_y \frac{d\omega_y}{dt} = \left[\left(\frac{2\pi^2 + 9}{9}\right)J\omega\ _x - \frac{8}{9}J\omega\ _y\right]\cos\gamma + Mgfl\cos\gamma$$

where Ji = (MR2/4) + Ml2 is the top's mass moment of inertia around axis i, J is the top's mass moment of inertia; other components are as specified above.

A tilted, well-balanced spinning top with a high angular velocity proves its capacity for self-stabilization. The axis of the tilted spinning top goes to the vertical position by the action of the inertial torques generated by the rotating mass elements, which values are bigger than torques generated by the top's weight and inertial torque generated by the centre mass. The necessary condition for a top's self-stabilization is formulated by separating variables of the torques produced by the centrifugal forces and the center mass of the top, and by the inertial torques generated by the rotating mass elements of the top. These two types of torques are represented on the right side of Eq. (1) and expressed by the following equation:

$$Mgl\cos\gamma + Ml^2\cos\gamma\sin\gamma\left[\left(\frac{2\pi^2 + 8 + (2\pi^2 + 9)\cos\gamma}{2\pi^2 + 9 - (2\pi^2 + 8)\cos\gamma}\right)\omega_x\right]^2 =$$
$$\left|-\left[\left(\frac{2\pi^2 + 8}{9}\right) + \left(\frac{2\pi^2 + 8 + (2\pi^2 + 9)\cos\gamma}{2\pi^2 + 9 - (2\pi^2 + 8)\cos\gamma}\right)\right]J\omega\omega_x - \left(\frac{9}{2\pi^2 + 9}\right)\left(\frac{2\pi^2 + 8 + (2\pi^2 + 9)\cos\gamma}{2\pi^2 + 9 - (2\pi^2 + 8)\cos\gamma}\right)Mgf\right|$$

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Analysis of Eq. (3) shows the equilibrium of the acting torques depends on four components, i.e. the mass and the angular velocity ω of the top, the velocity of precession ω_x, the angle γ of its inclination and the length of the top's leg. The stabilization process is intensive when the value of the inertial torques of the right side equation is big, and also, the length of the top's leg should be short, and i.e. the centre-mass of the top is located towards the tip of the leg. The spinning top with a long leg and a small radius of the disc manifests the less stability. The top of high spinning velocity generates the high value of the inertial torques represented on the right side of Eq. (3).

The obtained mathematical model for the top's motions enables for the describing physical principles of acting forces and the conditions of the top for self-stabilization. The application of new mathematical models for the top's motions effectively demonstrates the principles of the top's properties. In that regard, this is also a good example of educational processes for engineering mechanics.